

Statistical Test for Latent Growth Nonlinearity with Three Time Points*

By: Joseph F. T. Nese, University of Oregon

ABSTRACT

Curriculum-based measurement (CBM) is a system of assessment used to screen for students at risk for poor learning. CBM benchmark screening assessments are typically administered to all students in the fall, winter, and spring, and these data are frequently used by researchers to model and perhaps explain within-year growth. Modeling growth with three time points involves distinct choices in the functional form of growth that can be modeled as well as the parameters that can be estimated. Generally, only a linear growth model can be fully specified; however, research suggests that within-year CBM growth is often nonlinear, with decreasing or decelerating growth across the year. The purpose of this brief is to demonstrate how an estimated slope factor loading approach can be used in a structural equation modeling (SEM) framework to obtain a direct statistical test of nonlinearity provided by the output of your SEM program.

discrepancies in students' academic performance levels and trajectories and by providing a means by which to evaluate the effectiveness of instruction in producing academic growth.

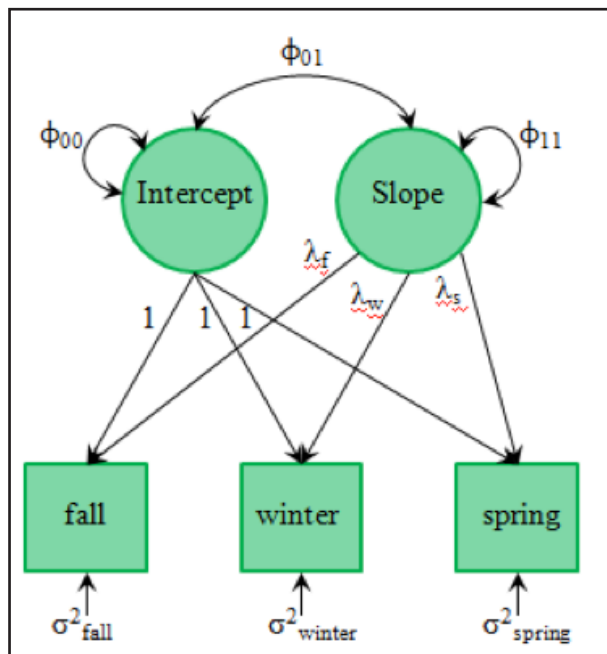
The focus of this brief is on CBM benchmark assessments, which are typically administered to all students in the fall, winter, and spring. These data are used by educators in an RTI framework to evaluate the academic status of groups or individual students, and by researchers to model and perhaps explain within-year growth. Modeling growth with three time points (fall, winter, and spring), however, involves distinct choices in the functional form (i.e., shape) of growth that can be modeled as well as the parameters that can be estimated. Here, we explore growth in a structural equation modeling (SEM) framework, as opposed to a multilevel framework (e.g., hierarchical linear modeling [HLM]), as the former offers more flexibility, particularly for evaluating and estimating nonlinear growth when only three time points are available. An SEM latent growth model with three time points is displayed in Figure 1.

As a rule, the number of possible growth parameters (including the intercept and linear slope, quadratic slope, cubic slope, etc.) is equal to the number of time points minus 1, if all parameters are to be estimated. So with three time points, only a linear model can be fully specified, such that the two growth components are the intercept and the linear slope parameters (see Figure 1). Alternatively, an additional slope parameter could be added to the model to represent either piecewise growth (gains from fall-winter and winter-spring) or quadratic growth (accelerating or decelerating), but with only three time points at least one variance/covariance component would need to be constrained in order for the model to be identified and thus possible to estimate. That is, to model either of these nonlinear models with three time points, one of

Curriculum-based measurement (CBM) is a system of assessment used to screen for students at risk for poor learning outcomes and monitor student progress to provide teachers with evidence related to the effectiveness of their instructional programs². CBM is an integral part of response to intervention (RTI), an educational model based on a multi-level prevention framework that involves assessment and effective intervention to increase student achievement. In practice, all students are administered universal screening (i.e., benchmark) assessments. Generally, those students who score below a pre-determined cut score on the benchmark assessment are then delivered specific instructional interventions that target an academic need, and are administered progress monitoring assessments to help teachers plan better instructional programs both by identifying

the following random effects must be fixed to zero: (a) the variance of the intercept factor or one of the two growth factors, applying the assumption that students do not vary in their initial achievement or rate of growth, respectively; or (b) the covariance between any two of the three parameters, applying the assumption that the two are orthogonal, or unrelated. If these assumptions render the research question untenable, an alternative is to freely estimate one of the slope factor loadings within an SEM framework. The purpose of this brief is to demonstrate how this approach can be used to offer a statistical test of nonlinearity.

Figure 1. Latent growth model with three time points.



the estimation of the spring slope factor loading (λ_f , λ_w , $\lambda_s = 0, 1, *$; where $*$ represents a freely-estimated parameter). The fall to winter gain, then, is represented by the mean of the slope factor; that is, the mean of the slope multiplied by the difference between the winter and fall slope factors, or 1 (i.e., 1 minus 0). The winter to spring gain is represented by the mean of the slope factor multiplied by the difference between the spring and winter slope factors. That is, the winter to spring slope estimate is a function of the estimated slope factor loading. In this example, if the estimated

spring slope factor loading is less than 2, the model demonstrates greater average fall to winter change than winter to spring change (i.e., more growth in the first half of the year than in the second); and if the estimated spring slope factor loading is greater than 2, the model demonstrates greater average winter to spring change than fall to winter change (i.e., more growth in the second half of the year than in the first).

Often, information criteria (e.g., Akaike's information criterion [AIC] and the Bayesian information criterion [BIC]) are used to compare the linear growth model and the estimated slope factor loading model to select the "best" model. Although smaller information criterion values indicate better fit, the magnitude is not directly interpretable. That is, although you can use the information criteria to determine which model better fits the data, the question remains: Is one model statistically better than the other? Or asked in a different way: Does the estimated slope factor loading model represent significant nonlinearity?

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Using the previous estimation of the spring slope factor loading as an example (λ_f , λ_w , $\lambda_s = 0, 1, *$), we can test whether the estimated factor loading is significantly different than 2. To do so, we subtract 2 from the estimated factor loading, and divide by the standard error of the estimated factor

Estimated Slope Factor Loading

As is general practice for a traditional SEM growth model, the intercept factor loadings are all fixed at 1 (see Figure 1). For a linear growth model with three time points, the slope factor loadings λ_f , λ_w , and λ_s are often fixed at (0, 1, 2), respectively. This puts the latent slope factor on a "seasonal" scale such that a one unit increase represents the growth from fall to winter and from winter to spring. Of course, this scale is determined by the researcher and can be placed, for example, on a monthly scale (e.g., 0, 4, 8) or weekly scale (e.g., 0, 18, 36). All of these slope factor loading examples specify the intercept to be at fall, and a constant rate of change across the year such that the growth from fall to winter is equal to that of the growth from winter to spring. Increasingly, however, within-year CBM growth is being described as nonlinear, generally with decreasing or decelerating growth across the year³. The estimated slope factor loading approach accommodates nonlinear growth and estimates all random parameters⁴, by freely estimating one of the slope factor loadings (λ_f , λ_w , or λ_s)⁵. This approach essentially offers gain score estimates, allowing the growth from fall-winter to be different than the growth from winter-spring. Take, for example,

loading. The absolute value of the resulting ratio can be looked up in a distribution table (e.g., z-table or t-table, depending on the SEM program you are using) to obtain the probability for values greater than or equal to the ratio; that is, the p-value of the difference between the estimated factor loading and 2. If the estimated time score is significant at the a priori alpha level (e.g., $p < .05$), we can infer that the nonlinear model is preferred because we have statistical evidence for nonlinear growth in the population data.

Perhaps a simpler approach is to estimate the fall slope factor loading (*, 1, 2). Thus, we now want to determine whether the estimated fall slope factor loading is different than zero. Rather than looking it up in a z-table, the output of your SEM program will offer a direct statistical test as a difference from zero, as it does with all estimated parameters. Then, if the estimated fall slope factor loading is negative and statistically different from zero, we know that there is greater change fall to winter change than winter to spring, and that this difference is statistically significant. If the estimated fall slope factor loading is positive and statistically different from zero, there is greater change winter to spring than fall to winter, and this difference is statistically significant. If the estimated fall slope factor loading is not significantly different from zero, we have statistical evidence to conclude the data are best represented by a linear model.

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⁴This model assumes the residual variances of the fall, winter, and spring assessments are constrained to be equal. Also note that the estimated slope factor loading approach is currently not possible in an HLM framework.

⁵As a rule at least two slope factor loadings must be fixed in a model with two growth factors, no matter how many time points, in order to scale the latent growth factor(s).