Testing and Interpreting Interaction Effects in Multilevel Models

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Presented at the annual AERA conference, Washington, DC, April, 2016

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Presentation available on NCAASE web site: http://www.ncaase.com/

This research was funded in part by a Cooperative Service Agreement from the Institute of Education Sciences (IES) establishing the National Center on Assessment and Accountability for Special Education – NCAASE (PR/Award Number R324C110004); the findings and conclusions expressed do not necessarily represent the views or opinions of the U.S. Department of Education.

Presentation Purpose

- Demonstrate analysis and interpretation of interactions in multilevel models (MLM)
  - Cross-level interactions of predictors at one level moderating growth parameters at a lower level
  - Product term interactions at same level and across levels
- Results of our studies of mathematics achievement growth for students with learning disabilities (LD) and general education (GE) students used as illustrations
  - Does LD status at level-2 interact with level-1 growth parameters (two-way, cross-level interaction)?
  - Do student socio-demographic characteristics interact with LD status?
  - Does the LD x Black race/ethnicity interaction at level-2 interact with level-1 growth parameters (three-way interaction)?

Cross-level Interactions in Multilevel Models

- While many MLM studies incorporate cross-level interactions, it is much less common for analysts to conduct complete post-hoc testing when interactions are significant

Level-1 Model: $Y_{ti} = \pi_{0i} + \pi_{1i}*(\text{Time}_{ti}) + \pi_{2i}*(\text{Time}^2_{ti}) + e_{ti}$ (1)

Level-2 Model: $\pi_{0i} = \beta_{00} + \beta_{01}*(\text{Predictor}_i) + r_{0i}$ (2)

$\pi_{1i} = \beta_{10} + \beta_{11}*(\text{Predictor}_i) + r_{1i}$ (3)

$\pi_{2i} = \beta_{20} + \beta_{21}*(\text{Predictor}_i) + r_{2i}$ (4)

Mixed Model: $Y_{ti} = \beta_{00} + \beta_{01}*\text{Predictor}_i + \beta_{10}*\text{Time}_{ti} + \beta_{11}*\text{Predictor}_i*\text{Time}_{ti} + \beta_{20}*\text{Time}^2_{ti} + \beta_{21}*\text{Predictor}_i*\text{Time}^2_{ti} + r_{0i} + r_{1i}*\text{Time}_{ti} + r_{2i}*\text{Time}^2_{ti} + e_{ti}$ (5)

Substantive Example: Interactions of Disability Status and Other Student Characteristics

- Many studies do not directly test the interaction of SWD status and other covariates thought to be related to student performance (e.g., LD status and sex of student)
- When these covariates are included as predictors (especially in regression and MLM models), only partial regression effects not the actual interactions are analyzed
- This can be very misleading and result in incorrect interpretations as well as incomplete understanding of group differences
- Interpretation also incomplete in MLM analyses when cross-level interactions are not probed and tested fully

Stevens, J. J., & Schulte, A. C. (2016). The interaction of learning disability status and student demographic characteristics on mathematics growth. *Journal of Learning Disabilities*. DOI: 10.1177/0022219415618496

Examples of Interaction Testing

- Student scores on the mathematics subtest of the Arizona Instrument to Measure Standards (AIMS) used to examine cross-level interaction of level-2 LD status with level-1 growth parameters (Grades 3 to 5)

- Student scores on the mathematics subtest of the North Carolina state test used to demonstrate three-way interaction of level-2 LD x Black race/ethnicity with level-1 growth parameters Grades 3 to 7)

- Details on sample, methods and procedures available in full papers

Cross-level Interaction with Level-1 Growth Parameters

- When a level-2 predictor (e.g., LD status) is used to predict growth at level-1, a two-way, cross-level interaction is formed
- If the cross-level interaction is statistically significant, post-hoc tests needed to determine specific differences (e.g., between GE vs. LD groups? From one grade to another?)
- Equivalent to "simple effects" and "simple slopes" post hoc tests in ANOVA

Two-level Linear MLM Growth Model: AIMS Data Grades 3 to 5

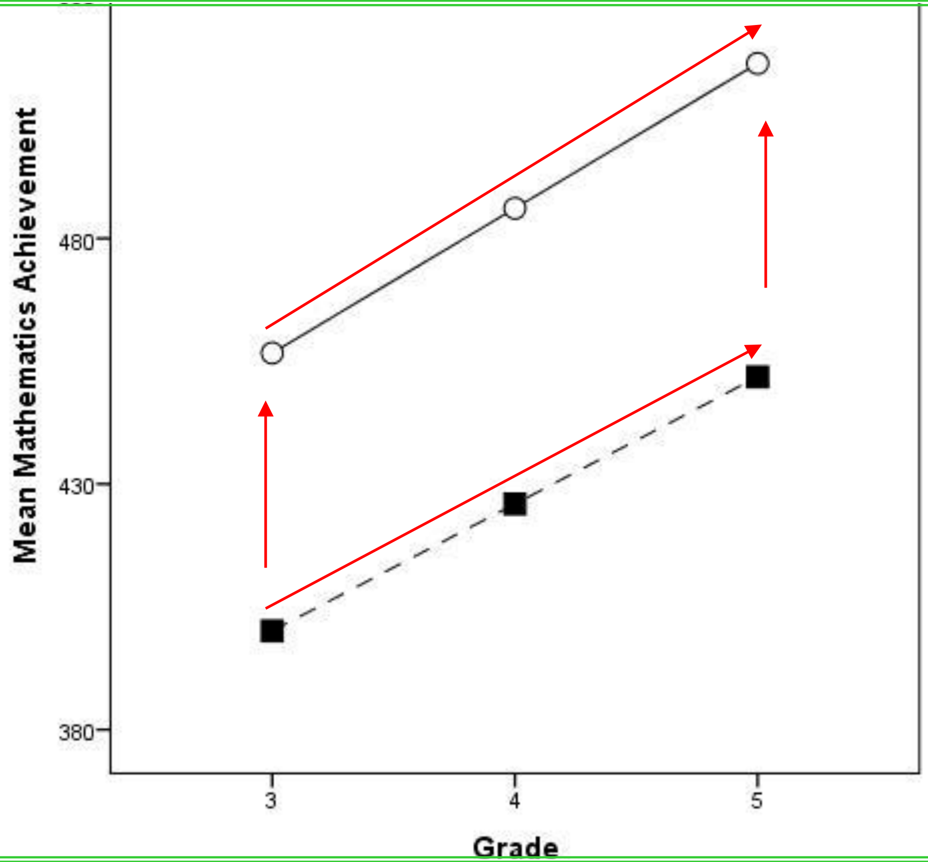
Fixed Effect	Coefficient	SE	t-ratio	df	p
Intercept, β_{00}	464.148472	0.186455	2489.331	75498	<0.001
LD, β_{01}	-58.878605	0.681264	-86.426	75498	<0.001
LEP, [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible].001
LD X [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible].001
Slope, β_{10}	29.396669	0.075204	390.893	75498	<0.001
LD, β_{11}	-3.574548	0.290520	-12.304	75498	<0.001

LD x Slope Cross-level Interaction

EB Means for the LD x Slope Cross-level Interaction

	Grade		
Group	3	4	5
GE	456.66 (39.92)	486.14 (42.79)	515.63 (45.72)
LD	400.10 (28.65)	425.95 (31.03)	451.81 (33.44)

Simple effects of slope for each separate group, horizontal" analysis within each group



Simple effects differences between the GE vs. LD trajectories, "vertical" analysis between groups at each time point

Simple effects of slope for each separate group

- Output provides test of simple slope for GE students, but need to test trajectory for LD students
- Simple effect of LD intercept or slope:

$$t = \beta_{LD} \ / \ SE_{\beta_{LD}}$$

- Where general formula for *SE* at moderator value M is:

$$SE_{\beta_{00LD}} = [SE^2(\beta_{00}) + (2M)\mathrm{cov}(\beta_{00}, \beta_{11}) + M^2 SE^2(\beta_{11})]^{½}$$

Note. *SE* formula above for either continuous or dichotomous predictors; simplifies for dichotomous predictors.

Simple effect of intercept or slope for each separate group

- With our dichotomous moderator, when M = 1, intercept *SE*:

$$SE_{\beta_{0,LD}} = [SE^2(\beta_{00}) + 2\mathrm{cov}(\beta_{00}, \beta_{01}) + SE^2(\beta_{01})]^{1/2}$$

- Slope SE:

$$SE_{\beta_{1,LD}} = [SE^2(\beta_{10}) + 2\mathrm{cov}(\beta_{10}, \beta_{11}) + SE^2(\beta_{11})]^{1/2}$$

Note. When M = 0, formulas simplify to $SE_{\beta_{0,GE}} = [SE^2(\beta_{01})]^{1/2}$ or $SE_{\beta 1,LD} = [SE^2(\beta_{01})]^{1/2}$.

Alpha Adjustment

- Repeated testing in post-hoc analysis can result in the inflation of Type I error (i.e., alpha slippage)
- We used Bonferroni's adjustment for post-hoc tests
- The nominal alpha level (.05) was divided by the number of tests within a family of comparisons (see Pedhazur, 1997, p. 435) to create a decision rule that took the number of comparisons tested into account

Simple Effects of GE vs. LD (Latent Class Analysis)

- Simple effect of GE vs. LD at selected time points (t):

$$\Delta_y = \beta_{01} + \beta_{11}(t)$$

$$t_{\Delta y} = \Delta_y \,/\, [SE^2(\beta_{01}) + (2t)\mathrm{cov}(\beta_{01}, \beta_{11}) + SE^2(\beta_{11})]^{1/2}$$

- When moderator is continuous, defines a "region of significance" where the two groups are significantly different (Potoff, 1964)

Example of MLM Three-way Interaction

- We were also interested in the product term interaction of student characteristics at level-2 (e.g., LD x Black race/ethnicity) and how those groups interacted with growth at level-1
- To accomplish this we computed the product of the LD and Black dummy codes and then used LD, Black and LD x Black as predictors in a two-level MLM of NC math achievement growth
- The predictors were used to model all random growth parameters (intercept, linear change, curvilinear change) over five grades

MLM Curvilinear Growth Model with LD x Black Interaction Effect

Fixed Effect	Coefficient	*SE*	*t*	*df*	*p*
Intercept, β_{00}	253.857622	0.040764	6227.510	79544	<0.001
LD, β_{04}	-4.659241	0.110734	-42.076	79544	<0.001
BLACK, β_{06}	-4.401213	0.055501	-79.299	79544	<0.001
LDxBLACK, β_{09}	0.425137	0.194290	2.188	79544	0.029
Linear Slope, β_{10}	7.015400	0.024868	282.103	79544	<0.001
LD, β_{14}	-0.706862	0.071533	-9.882	79544	<0.001
BLACK, β_{16}	0.221137	0.035939	6.153	79544	<0.001
LDxBLACK, β_{19}	-0.405060	0.138214	-2.931	79544	0.003
Curvilinear, β_{20}	-0.526089	0.006246	-84.226	79544	<0.001
LD, β_{24}	-0.008205	0.017716	-0.463	79544	0.643
BLACK, β_{26}	-0.111824	0.008944	-12.502	79544	<0.001
LDxBLACK, β_{29}	0.105315	0.034352	3.066	79544	0.002

Note. Table presented for illustrative purposes. Complete results available in Stevens & Schulte (2016).

Conducting Statistical Tests

- Process is parallel to AIMS analysis above:
  - Bonferroni-adjusted simple slope effects; each of the four interaction groups' growth trajectories (see Figure below) calculated by rotating coding of the dichotomous predictors as described above
  - Simple effect group differences, also a direct extension of presentation above (equivalent to a LCA with 3-way interactions)
  - We also calculated pairwise comparisons of the four interaction groups at each point in time (Grade) to allow specific tests of group differences at each grade

Pairwise Comparisons of Group Differences at Each Grade

- "Vertical" comparisons of groups at each point in time:

$$t = (\beta_{LDt} - \beta_{Blackt}) / SE_{Group}$$

$$SE_{Group} = [SE^2(\beta_{LD}) + SE^2(\beta_{Black}) - 2\text{cov}(\beta_{LD}, \beta_{Black})]^{½}$$

Level-2 Interaction Means

- In the MLM regression equation (LD, Black, and LD x Black, respectively), a 2 x 2 matrix of the interaction group means at each grade is:

		LD Status	
		GE, 0	LD, 1
Race/ ethnicity	White, 0	β_0	$\beta_0 + \beta_{LD}$
	Black, 1	$\beta_0 + \beta_{BL}$	$\beta_0 + \beta_{LD} + \beta_{BL} + \beta_{LDxBL}$

There are six possible pairwise comparisons among these four interaction means (k[k-1]/2 = 4[3]/2 = 6)

Table of 3-Way EB Interaction Means

Group	Grade				
	3	**4**	**5**	**6**	**7**
GE-White	253.86	260.35	265.78	270.17	273.50
GE-Black	249.46	256.06	261.38	265.42	268.20
LD-White	249.20	254.97	259.68	263.32	265.88
LD-Black	244.80	250.68	255.27	258.57	260.58

Six pairwise comparisons at each grade for each growth parameter. For example, at Grade 3 (wave 1), six comparisons of the four group intercepts (*SE* = 0.3328)

	LD-Black	LD-White	GE-Black
GE-White	9.06	4.66	4.40
GE-Black	4.66	0.26 **ns**	
LD-White	4.40		

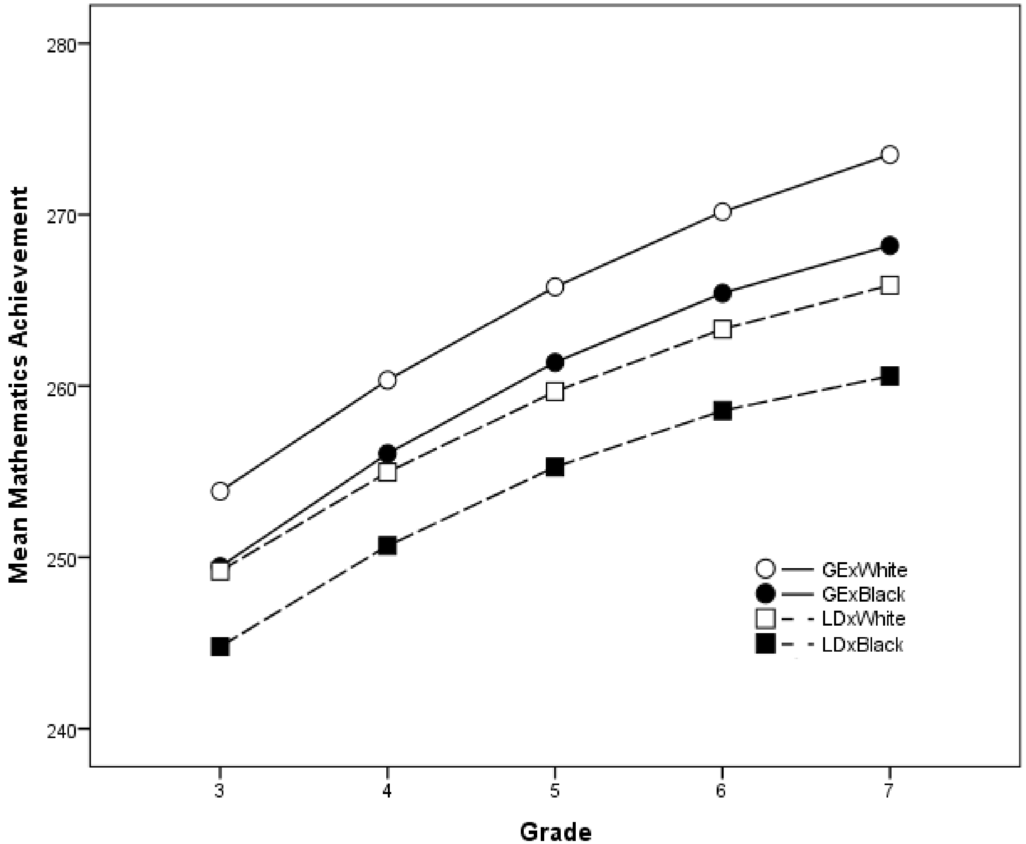


Figure 2. Three-way interaction of LD status, black-white race/ethnicity, and grade for the North Carolina sample.

Brief Bibliography

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Stevens, J. J., & Schulte, A. C. (2016). The interaction of learning disability status and student demographic characteristics on mathematics growth. *Journal of Learning Disabilities*. Advance online publication. doi: 10.1177/0022219415618496

Software Support

PROCESS software:

Hayes, A. F. (2013). *Introduction to mediation, moderation, and conditional process analysis: A regression-based approach*. New York, NY: Guilford Press.

http://afhayes.com/spss-sas-and-mplus-macros-and-code.html

Kristopher J. Preacher, interactive calculation tools for probing interactions in multiple linear regression, latent curve analysis, and hierarchical linear modeling:

http://www.quantpsy.org/interact/

Appendices

- Comparison of partial and interaction effects
- HLM screens for obtaining variance-covariance matrix output
- variance-covariance matrix output for the HLM two-level AIMS model with LD status at level-2

Partial Effects vs. Interaction Effects

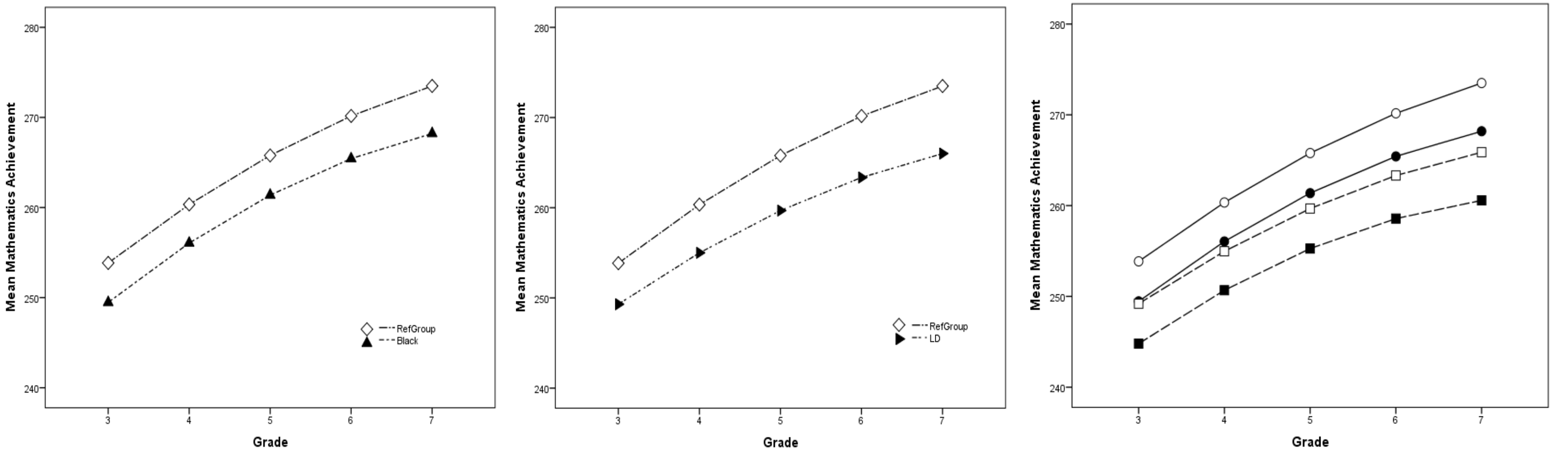
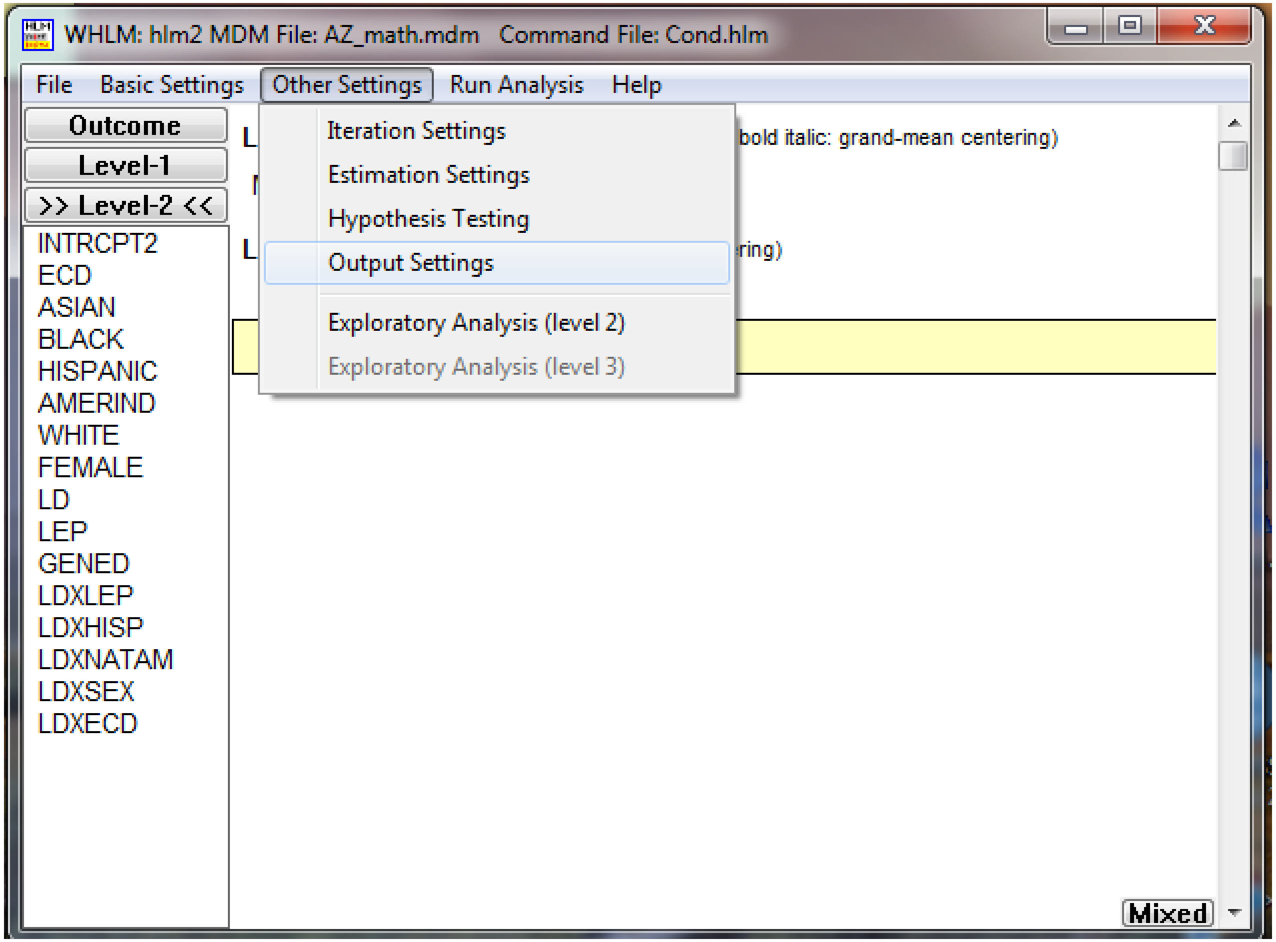
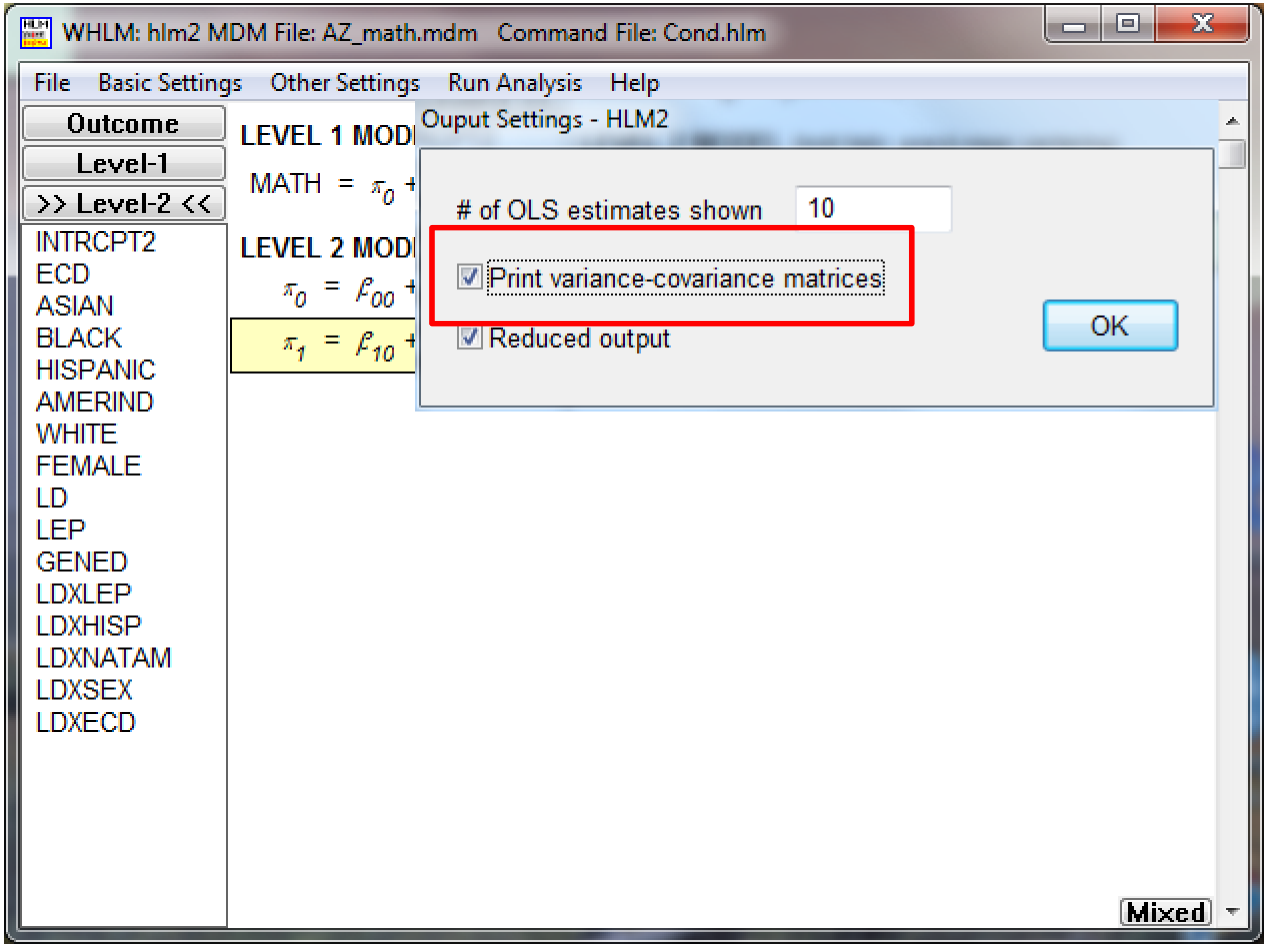


Figure 1. Partial regression effects with the reference group (intercept) displayed in each panel and the partial effect of Black race/ethnicity on the left, LD status in the middle, and the LD x Black interaction effect on right.

HLM Screens Showing Request for Variance-covariance Matrix Output




WHLM: hlm2 MDM File: AZ_math.mdm Command File: Cond.hlm
File
Basic Settings
Other Settings
Run Analysis
Help
Outcome
Level-1
>> Level-2 <<
INTRCPT2
ECD
ASIAN
BLACK
HISPANIC
AMERIND
WHITE
FEMALE
LD
LEP
GENED
LDXLEP
LDXHISP
LDXNATAM
LDXSEX
LDXECD
LEVEL 1 MODEL
LEVEL 2 MODEL
Ouput Settings - HLM2
# of OLS estimates shown
10
Print variance-covariance matrices
Reduced output
OK
Mixed

V-C Matrix Output for the AIMS Cross-level Interaction of LD Status with Intercept and Slope

β00	LD, β01	β10	LD, β11
456.6501997	-56.5625933	29.5123039	-3.6502561

β00	3.1762152E-002			
LD, β01	-3.1762152E-002	3.2058468E-001		
β10	-2.3881150E-003	2.3881150E-003	4.7033040E-003	
LD, β11	2.3881150E-003	-3.8899123E-002	-4.7033040E-003	5.7435505E-002

	r0	r1	LD0	LD1
r0	1712.9542784			
r1	121.9136308	13.8159393		
LD0	134.5603803	-13.7552066	4.7080326	
LD1	-13.7552066	12.2251105	-3.7933373	5.0056443

Level-1,e 563.5544473